Proceedings of IAM, V.9, N.1, 2020, pp.13-17

PITCHFORK DOMINATION AND ITS INVERSE FOR COMPLEMENT GRAPHS

Mohammed A. Abdlhusein[1], Manal N. Al-Harere[2]

[1] Department of Mathematics, College of Education for Pure Sciences (Ibn Al-Haitham),
Baghdad University, Baghdad, Iraq
[1] Department of Mathematics, College of Education for Pure Sciences,
Thi-Qar University, Thi-Qar, Iraq
[2]Department of Applied Sciences, University of Technology, Baghdad, Iraq
[1]Mmhd@utq.edu.iq
[2]100035@uotechnology.edu.iq

Abstract. Let $G = (V, E)$ be a finite simple and undirected graph without isolated vertices. A subset D of V is a pitchfork dominating set if every vertex $v \in D$ is dominates at least j and at most k vertices of $V - D$, for any non-negative integers j and k. A subset D^{-1} of $V - D$ is an inverse pitchfork dominating set with respect to D if it is a dominating set. The domination number of G, denoted by $\gamma_{pf}(G)$ is a minimum cardinality over all pitchfork dominating sets in G. The inverse domination number of G, denoted by $\gamma_{pf}^{-1}(G)$ is a minimum cardinality over all inverse pitchfork dominating sets in G. In this paper, an applying of pitchfork domination and it's inverse is given on some complement graphs when $j = 1$ and $k = 2$. Evaluations and proofs for $\gamma_{pf}(G)$ and $\gamma_{pf}^{-1}(G)$ of the complement graphs are given.

Keywords: pitchfork domination, inverse pitchfork domination, complement graph.

AMS Subject Classification: 05C69.

1. Introduction.

Let $G = (V, E)$ be a graph without isolated vertices has a vertex set V of order n and an edge set E of size m. For any vertex $v \in V$, the degree of v is defined as the number of edges incident on v and denoted by $deg(v)$. The minimum and maximum degrees of vertices denoted by $\delta(G)$ and $\Delta(G)$, respectively. The subgraph of G induced by the vertices in D is denoted by $G[D]$. The complement $\overline{G}$ of a simple graph G with vertex set $V(G)$ is the graph in which two vertices are adjacent if and only if they are not adjacent in G. For graph theoretic terminology we refer to [8]. The study of domination and related subset problems is one of the fastest growing areas in graph theory. For a detailed survey of domination one can see [4] and [5]. A set $D \subseteq V$ is a dominating set if every vertex in $V - D$ is adjacent to a vertex in D, that is $N[D] = V$. A dominating set D is said to be a minimal dominating set if no proper subset of D is a dominating set. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set D of G.

The importance of domination in various applications, led to the appearance of different types domination according to the purpose used for see for example [2, 3, 6, 7]. A new model of domination in graphs called the pitchfork domination and it's inverse are introduced by Al-harere and Abdlhusein [1, 2]. Here, a new application of this domination types are introduced and applied on some complement graphs.

Observation 1.1 *[2]: For a path graph P_n and cycle graph C_n, we have:* $\gamma_{pf}(P_n) = \gamma(P_n) = \gamma_{pf}(C_n) = \gamma(C_n) = \lceil \frac{n}{3} \rceil$.

Proposition 1.2 *[2] Let $G = K_n$ the complete graph with $n \geq 3$, then $\gamma_{pf}(K_n) = n - 2$.*

Note 1.3 *[2] For any graph G of order n and pitchfork domination number γ_{pf}, if $\gamma_{pf}(G) > \frac{n}{2}$ then G has no inverse pitchfork domination.*

2. Pitchfork Domination for Complement Graphs.

In this section, the pitchfork domination is studied on some complement graphs such as the complement of path, cycle, wheel, complete and complete bipartite graph.

Theorem 2.1. *Let P_n be a path graph, then:*

$$\gamma_{pf}(\overline{P}_n) = \begin{cases} 2, & if \;\; n = 4, \; 5, \; 6 \\ 3, & if \;\; n = 7 \\ n-2, & if \;\; n \geq 8 . \end{cases}$$

where $\overline{P}_n$ has no pitchfork domination for $n = 2, 3$.

Proof. Since $\overline{P}_2$ is a null graph and $\overline{P}_3$ has an isolated vertex, then $\overline{P}_2$ and $\overline{P}_3$ has no pitchfork domination. let us label the vertices of $\overline{P}_n$ as $\{v_i;\; i = 1, 2, \dots, n\}$. If $n = 4$ let D has any two vertices except $\{v_i, v_{i+2}\}$ because these vertices don't adjacent v_{i+1} vertex. If $n = 5$ let $D = \{v_1, v_5\}$. If $n = 6$ let $D = \{v_2, v_5\}$. If $n = 7$ let $D = \{v_i, i\ is\ even\}$. Now if $n \geq 8$, let $D = \{v_2, v_3, \dots, v_{n-1}\}$ and $V - D = \{v_1, v_n\}$ where D dominate all the vertices of $\overline{P}_n$ where the vertices $v_3, v_4, \cdots, v_{n-2}$ dominate v_1 and v_n while v_2 dominates only v_n, also v_{n-1} dominates only v_1. In all above cases, D is a minimum pitchfork dominating set. Thus D is a γ_{pf} −set of $\overline{P}_n$.

Theorem 2.2. *Let C_n be a cycle graph of order $n \geq 3$, then:*

$$\gamma_{pf}(\overline{C}_n) = \begin{cases} \lceil \frac{n}{3} \rceil, & if \;\; n = 4, \; 5, \; 6 \\ n-4, & if \;\; n = 7, \; 8 \\ 6\,, & if \;\; n = 9 \\ n-2, & if \;\; n \geq 10 . \end{cases}$$

Proof. There are some cases to find the pitchfork domination number, where $\overline{C}_n$ is r −regular graph with $r = n - 3$. If $n = 3$ then $\overline{C}_3$ has no pitchfork domination since it is a null graph. If $n = 4$ since $\overline{C}_4$ disconnected graph has two P_2 components and since $\gamma_{pf}(P_2) = 1$ from Observation (1.1), then $\gamma_{pf}(\overline{C}_4) = 2$. If $n = 5$ let D consists of any two consecutive vertices. If $n = 6$ let $D = \{v_k, v_{k+3}\}$ for any integer $1 \leq k \leq 6$. If $n = 7$ let us label the vertices of $\overline{C}_7$ as $\{v_i;\ i = 1, 2, \cdots, 7\}$ and let $D = \{v_i;\ i\ is\ odd,\ \ i \neq 7\}$. If $n = 8$ let $D = \{v_i;\ i\ is\ odd\}$. If $n = 9$ let D consists of the first two vertices from every three consecutive vertices. If $n \geq 10$ let D consists of all the vertices except two vertices, but we must avoid choose $V - D = \{v_i, v_{i+2}\}$ since v_{i+1} don't dominates any vertex. In all previous cases, D is a minimum pitchfork dominating set. Hence D is a γ_{pf} −set of $\overline{C}_n$.

Theorem 2.3. *Let $K_{n,m}$ be a bipartite graph such that $n \neq 1$, then:*

$$\gamma_{pf}(\overline{K}_{n,m}) = \begin{cases} 2, & if\ \ n = m = 2 \\ n + m - 3, & if\ \ n = 2, \quad\ m \geq\ 3 \\ n + m - 4, & if\ \ n, \quad\ m \geq\ 3\ . \end{cases}$$

Proof. Since $\overline{K}_{1,m}$ has an isolated vertex, then it has no pitchfork domination. Since $\overline{K}_{n,m}$ is a disconnected graph contains two components H_1 and H_2 of order n and m respectively. Thus if $n = m = 2$ then every component is a P_2 graph with $\gamma_{pf}(P_2) = 1$ according to Observation (1.1). If $n, m \geq 3$ then every component H_i is a complete graph their pitchfork domination evaluated according to Proposition (1.2) and equal $|H_i| - 2$. If $n = 2, m \geq 3$ then we can combine the above two cases. In all previous cases, D is a minimum pitchfork dominating set. Hence D is a γ_{pf} −set of $\overline{K}_{n,m}$.

Proposition 2.4. *Let K_n be a complete graph, and W_n be a wheel graph, then $\overline{K}_n$ and $\overline{W}_n$ has no pitchfork domination.*

Proof. Since $\overline{W}_n$ has an isolated vertex and $\overline{K}_n$ is a null graph.

3. An Inverse Pitchfork Domination.

Here, the inverse pitchfork domination is applied on some complement graphs which was studied on the previous section. Where we choose the inverse pitchfork dominating set with respect to the pitchfork dominating set in the same graph.

Theorem 3.1. *Let P_n be a path graph, then $\overline{P}_n$ has an inverse pitchfork domination if and only if $n = 4, 5, 6, 7$ such that:*

$$\gamma_{pf}^{-1}(\overline{P}_n) = \begin{cases} 2, & if \;\; n = \; 4, \;\; 5 \\ n-3, & if \;\; n = \; 6, \;\; 7 \end{cases}$$

Proof. let us label the vertices of $\overline{P}_n$ as $\{v_i;\; i = 1, 2, \dots, n\}$. If $n = 4, 5$ let D^{-1} has any two vertices of $V - D$ such that $D^{-1} \neq \{v_i, v_{i+2}\}$. If $n = 6$ let D^{-1} has any three vertices of $V - D$. If $n = 7$ then $D^{-1} = V - D$. Thus D^{-1} is an inverse pitchfork dominating set, then D^{-1} is a γ_{pf}^{-1} −set of $\overline{P}_n$. If $n \geq 8$, since $\gamma_{pf}(\overline{P}_n) = n - 2 > \frac{n}{2}$, then $\overline{P}_n$ has no inverse pitchfork domination according to Note (1.3).

Theorem 3.2. *Let C_n be a cycle graph of order $n \geq 3$, then $\overline{C}_n$ has an inverse pitchfork domination if and only if $4 \leq n \leq 8$, such that:*

$$\gamma_{pf}^{-1}(\overline{C}_n) = \begin{cases} \lceil \frac{n}{3} \rceil, & if \;\; n = 4, \;\; 5, \;\; 6 \\ n-4, & if \;\; n = 7, \;\; 8\,. \end{cases}$$

Proof. According to the minimum pitchfork dominating set D in Theorem (2.2), let us choose D^{-1} as follows: If $n = 4$ then let $D^{-1} = V - D$. If $n = 5$ then let D^{-1} has any two consecutive vertices from $V - D$. If $n = 6$ then let $D^{-1} = \{v_{k+1}, v_{k+4}\}$ for any integer $1 \leq k \leq 6$. If $n = 7, 8$ then let $D^{-1} = \{v_i;\; i\; is\; even\}$. D^{-1} is an inverse pitchfork dominating set, then D^{-1} is a γ_{pf}^{-1} −set of $\overline{C}_n$. If $n \geq 9$ then $\overline{C}_n$ has no inverse pitchfork domination by Note (1.3) since $\gamma_{pf}(\overline{C}_n) > \frac{n}{2}$.

Theorem 3.3. *Let $K_{n,m}$ be a bipartite graph such that $n \neq 1$, then $\overline{K}_{n,m}$ has an inverse pitchfork domination if and only if $n, m = 2, 3, 4$ such that:*

$$\gamma_{pf}^{-1}(\overline{K}_{n,m}) = \begin{cases} 2, & if \;\; n = m = 2 \\ n+m-3, & if \;\; n = 2, \qquad m = \;\; 3, \;\; 4 \\ n+m-4, & if \;\; n, \;\; m = \;\; 3, \;\; 4\,. \end{cases}$$

Proof. $\overline{K}_{n,m}$ is a disconnected graph contains two components H_1 and H_2 of order n and m respectively. Thus any component of order two is a P_2 graph with $\gamma_{pf}^{-1}(P_2) = 1$. If the order of any component equals three or four then it is a complete graph has an inverse pitchfork domination equals $|H_i| - 2$. If $|H_i| > 4$ then $\overline{K}_{n,m}$ has no inverse pitchfork domination by Note (1.3) since $\gamma_{pf}(\overline{K}_{n,m}) > \frac{n+m}{2}$.

Note 3.4. *According to the above results, we have:*

1. $\gamma_{pf}(\overline{P}_n) + \gamma_{pf}^{-1}(\overline{P}_n) = n$ if and only if $n = 4, 7$.
2. $\gamma_{pf}(\overline{C}_n) + \gamma_{pf}^{-1}(\overline{C}_n) = n$ if and only if $n = 4, 8$.

3. $\gamma_{pf}(\overline{K}_{n,m}) + \gamma_{pf}^{-1}(\overline{K}_{n,m}) = n + m$ if and only if $n = 2, 3, 4$.

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